

B.Sc. Semester - 1 (NEP-2020) Examination
OCT/NOV-2025 (As Per Syllabus Year June-2025)
MATH: .Matrices and Coordinate Geometry (Major-1)

Time: 2:00 Hours

Marks:50

Instructions:

1. All questions are compulsory.
2. Figures to the right indicate marks.

Que.1(A) Answer any one. (05)

- (1) Define: Inverse of a matrix. If A and B are non-singular matrices then prove that :
 (i) $(AB)^{-1} = B^{-1}A^{-1}$ (ii) A^T is non-singular and $(A^T)^{-1} = (A^{-1})^T$
- (2) Show that every square matrix is uniquely expressible as the sum of a Hermitian and a skew-Hermitian matrix.

Que.1(B) Answer any one. (05)

- (1) Define: Involutory matrix with illustration.
 If $\begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix} A \begin{bmatrix} -3 & 2 \\ 5 & -3 \end{bmatrix} = \begin{bmatrix} -2 & 4 \\ 3 & -1 \end{bmatrix}$ then find A.
- (2) Define: Involutory matrix with illustration. Prove that:
 A is involutory iff $(I + A)(I - A) = 0$.

Que.2(A) Answer any one. (05)

- (1) Find Inverse of $A = \begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$ by using elementary row operations.
- (2) Define: Rank of a matrix. Find the rank of $A = \begin{bmatrix} 8 & 1 & 3 & 6 \\ 0 & 3 & 2 & 2 \\ -8 & -1 & -3 & 4 \end{bmatrix}$ using echelon form.

Que.2(B) Answer any one. (05)

- (1) Prove that row rank of a matrix is the same as its rank.
- (2) Find two non-singular matrices P and Q such that PAQ is in the normal form, where $A = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 2 & 3 \\ 0 & -1 & -1 \end{bmatrix}$.

Que.3(A) Answer any one. (05)

- (1) State and prove Cayley-Hamilton theorem for a square matrix.
- (2) Using Cayley-Hamilton theorem, find the inverse of the matrix
 $A = \begin{bmatrix} 2 & 1 & -1 \\ 0 & 2 & 1 \\ 5 & 2 & -3 \end{bmatrix}$.

Que.3(B) Answer any one. (05)

- (1) Define: Eigen values and Eigen vectors. Prove that Eigen values of Hermitian matrix are real numbers.
- (2) Solve: $5x + 3y + 7z = 4, 3x + 26y + 2z = 9, 7x + 2y + 10z = 5$.

Que.4(A) Answer any one. (05)

(1) Convert the following Cartesian co-ordinates into polar form.

(i) $(\sqrt{3}, -1)$ (ii) $\left(-\frac{\sqrt{3}}{2}, -\frac{\sqrt{3}}{2}\right)$ (iii) $(-1, 1)$ (iv) $(0, 1)$

(2) Find the distance between the following points.

(i) $(2, 40^\circ)$, $(4, 100^\circ)$ (ii) $(-3, 45^\circ)$, $(7, 105^\circ)$

Que.4(B) Answer any one. (05)

(1) Find the condition for two lines $p = r\cos(\theta - \alpha)$ and $p' = r\cos(\theta - \alpha')$ to be perpendicular.

(2) Obtain the equation of the circle whose end points at the diameter are $A(r_1, \theta_1)$ and $B(r_2, \theta_2)$.

Que.5(A) Answer any one. (05)

(1) Define: Periodic matrix. Find the period of matrix $\begin{bmatrix} -1 & -1 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$.

(2) Prove that the rank of the product of two matrices cannot exceed the rank of either matrix.

Que.5(B) Answer any one. (05)

(1) Solve: $x + y + z + w = 0$, $x + 3y + 2z + 4w = 0$, $2x + z - w = 0$.

(2) Obtain Cartesian and spherical co-ordinates for the cylindrical co-ordinates $(6, 120^\circ, -2)$.
